# Electron-phonon interaction in a two-subband quasi-2D system in a quantizing magnetic field

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We predict a double-resonant feature in the magnetic field dependence of the phonon-mediated mobility of a two-subband quasi-two-dimensional electron system. These resonances take place when two Landau levels corresponding to different size-quantized subbands are close to each other but do not coincide. We also discuss the effect of non-equilibrium phonons. Rabi-like oscillations of electron population and emission of the phonons at new frequencies are predicted.

The specific feature of the two-dimensional (2D) electron system in a strong magnetic field is its effective zero dimension due to Landau quantization of the in-plane electron motion. This results in a strong suppression of electron-acoustic phonon scattering because it is impossible for dispersionless particles to fulfill simultaneously the energy and momentum conservations under emission or absorption of a single phonon and only the processes of the second order in electron-phonon interaction are allowed [1].

The different situation occurs when the cyclotron energy is close to the intersubband splitting, where the subband structure is formed by the quantization of electron motion in the direction perpendicular to a confinement plane. In this case the second Landau level of the first subband is close in energy to the first Landau level of the second subband. When the energy separation between these levels becomes of the order of $\Delta \sim \hbar s/l$ the transitions of electrons between the levels with emission or absorption of a single phonon become allowed, where $s$ is the speed of sound and $l$ is the magnetic length. Such processes should increase the dissipation conductivity, $\sigma_{xx}$. In what follows we calculate $\sigma_{xx}$ to the first order in electron-phonon interaction and show that it has a double peak structure as a function of level splitting or a magnetic field.

The above effect can be observed only in the system with electron filling factor greater than unity. Here and in what follows we neglect electron spins and consider a spinless system. We restrict our analysis to the case of the relatively low occupancy of the intersecting levels so that we can consider electrons in these levels as the non-interacting particles. The high-occupancy case, in which the many-body effects become essential will be considered somewhere else [2]. The many-electron effects in magneto-optics of two-subband system were studied both theoretically and experimentally in Ref. [3].

In a strong magnetic field the completely occupied first Landau level of the first subband can be considered as a non-dynamical background. Then the electron system can be mapped onto a two level system, where the first and the second levels are the second Landau level of the first subband and the first Landau level of the second subband, respectively. The splitting $\Delta$ between the levels is equal to $\Delta = \Delta_{01} - \hbar\omega_c$, where $\Delta_{01}$ is the intersubband splitting and $\hbar\omega_c$ is the cyclotron energy. For convenience we consider positive $\Delta$ only, the generalization to the case of negative $\Delta$ is straightforward.

We use the isotropic Debye approximation with a linear dependence of the phonon frequency on its wave vector: $\omega_j(Q) = s_j Q$, where $j$ is labeling the phonon mode, $j = 1$ for longitudinal and $j = 2, 3$ for two transverse modes, $s_j$ is the speed of sound of the $j$th mode.

The electron-phonon Hamiltonian can be written in the form [4]:

$$H_{e-ph} = -\sum_{j,\vec{Q}} \frac{M_j(\vec{Q})}{\sqrt{V}} Z(q_z) \left[ \hat{\rho}^+(\vec{q})\hat{d}_j^+(\vec{Q}) + \hat{\rho}(\vec{q})\hat{d}_j(\vec{Q}) \right] \quad , \tag{1}$$

where the capital letters $(\vec{Q})$ mean the 3D vectors, their projections onto the 2D layer are shown by corresponding small letters $(\vec{q})$; $\hat{d}_j^+$ is the creation operator of a phonon in the $j$th mode, $V$ is a normalization volume, $\hat{\rho}^+(\vec{q})$ is the electron density operator; $M_j(\vec{Q})$ are the matrix elements of electron-phonon interaction. In GaAs these matrix elements have the form [4]:

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$$M_j(\vec{Q}) = \sqrt{\frac{\hbar}{2\rho_0 sQ}} \left[ -\frac{eh_{14}}{\kappa\epsilon_0} \frac{Q_x Q_y \xi_{j,z} + Q_y Q_z \xi_{j,x} + Q_z Q_x \xi_{j,y}}{Q^2} - i\Xi_0(\vec{\xi}_j \cdot \vec{Q}) \right] , \tag{2}$$

where $\rho_0$ is the GaAs mass density, $h_{14}$ and $\Xi_0$ are the parameters of piezoelectric and deformation potential couplings [5], $\vec{\xi}_j$ is the polarization vector of the $j$th phonon mode. The form factor $Z(q_z)$ is determined by the electron spreading in $z$ direction:

$$Z(q_z) = \int dz e^{iq_z z} \chi_1(z)\chi_2(z) , \tag{3}$$

where $\chi_1(z)$ and $\chi_2(z)$ are the wave functions associated with the first and second subbands, respectively. We use the Fang-Howard approximation [6] for these functions:

$$\chi_1(z) = \sqrt{\frac{b^3}{2}} z \exp\left(-\frac{1}{2}bz\right) , \quad \chi_2(z) = \sqrt{\frac{b^5}{6}} z \left(z - \frac{3}{b}\right) \exp\left(-\frac{1}{2}bz\right) .$$

The dissipative conductivity can be found from the equation:

$$\sigma_{xx} = \frac{e^2}{2\pi l^2}\nu_1(1-\nu_2)\frac{D_{12}}{kT} + \frac{e^2}{2\pi l^2}\nu_2(1-\nu_1)\frac{D_{21}}{kT} = 2\frac{e^2}{h}\nu_1(1-\nu_2)\frac{\hbar}{l^2}\frac{D_{12}}{kT} , \tag{4}$$

where $\nu_1$ and $\nu_2 = \nu - \nu_1$ are the filling factors of the first and second levels, respectively; $\nu$ is the total filling factor; $T$ is the temperature; $D_{ij}$ is the electron diffusion coefficient corresponding to the phonon assisted transition of the electron from level $i$ to level $j$. In expression (4) the condition of thermal equilibrium $\nu_1(1-\nu_2)D_{12} = \nu_2(1-\nu_1)D_{21}$ was used. The diffusion coefficient is given by

$$\begin{aligned}
D_{12} &= \frac{l^4}{2}\left\langle \delta q^2 \right\rangle \\
&= \frac{l^4}{2}\frac{2\pi}{\hbar}\sum_j \int \frac{d\vec{Q}}{(2\pi)^3}\delta(\Delta - \hbar sQ)n_j(\vec{Q})q^2 \left|M_j(\vec{Q})Z(q_z)\right|^2 R_{01}(q) ,
\end{aligned} \tag{5}$$

where $n_j(Q)$ is the phonon distribution function, and $R_{01}(q) = \frac{(lq)^2}{2}e^{-(lq)^2/2}$. Equations (4) and (5) are derived using the arguments similar to those in Ref. [7]. The filling factors $\nu_1$ and $\nu_2$ are determined from the standard Fermi distribution for the non-interacting two-level system:

$$\nu_1 = \frac{\nu}{\nu + e^{-\Delta/2kT}\left[\xi + \sqrt{2 - \nu + \xi^2}\right]} , \quad \xi = (1-\nu)\cosh(\Delta/2kT) ,$$

and $\nu_2 = \nu - \nu_1$.

Substituting equations (2), (3) and (5) into (4) and performing the integration we get the dissipative conductivity in the form:

$$\begin{aligned}
\frac{\sigma_{xx}}{e^2/h} &= \nu_1(1-\nu_2)q_1^7 n(q_1)\int_0^1 du(1-u^2)^2 \left|Z(uq_1)e^{-q_1^2(1-u^2)/2}\right|^2 \times \\
&\quad \times \left[M_d + \frac{9}{8}\frac{M_{p,1}}{q_1^2}u^2(1-u^2)\right] + \nu_1(1-\nu_2)q_2^5 n(q_2)\frac{M_{p,2}}{8} \times \\
&\quad \times \int_0^1 du(1-u^2)^3(9u^4 - 2u^2 + 1)\left|Z(uq_2)\right|^2 e^{-q_2^2(1-u^2)/2} ,
\end{aligned}$$

where $q_1 = \Delta/\hbar s_1$, $q_2 = \Delta/\hbar s_2$ and constants $M_d$, $M_{p,1}$ and $M_{p_2}$ are given by

$$M_d = \frac{1}{8\pi}\frac{\Xi^2}{kT\rho_0 s_1^2 l^3} , \quad M_{p,1} = \frac{1}{8\pi}\frac{e^2 h_{14}^2 \hbar}{kT\rho_0 s_1 \kappa^2\epsilon_0^2 l} , \quad M_{p,2} = \frac{1}{8\pi}\frac{e^2 h_{14}^2 \hbar}{kT\rho_0 s_2 \kappa^2\epsilon_0^2 l} .$$

The typical double peak dependence of the dissipative conductivity $\sigma_{xx}$ is shown in Fig.1a for $l(B_0) = 50\text{Å}$, $b = 1/l(B_0)$ and two values of the temperature: $T = 3K$ (dotted line) and $T = 10K$ (solid line), where we assume that the levels coincide at magnetic field $B_0$. The conductivity is shown for $\nu = 0.1$ and it is normalized by $\nu e^2/h$.

In these units $\sigma_{xx}$ has a weak dependence on the total electron filling factor. The two-peak structure is clearly seen in the figure. The heights of the peaks increase with increasing the temperature which results from increasing the equilibrium phonon density. The described double peak structure should also be observed in the phonon specroscopy [8,9] experiments.

We would also like to discuss another phonon-mediated effect, which could be observed in the double-subband system under the influence of non-equilibrium ballistic phonons with the given direction of momentum $\vec{e}_q = \vec{q}/q$. This system is in fact a two-level system, in which the non-equilibrium phonons cause inter-level transitions between the states with the momentum difference $q_0 \vec{e}_{q,||}$. Here $q_0 = \Delta/\hbar s$, $\vec{e}_{q,||}$ is the projection of $\vec{e}_q$ on the 2D plane and we use the Landau gauge with $y$-axis alone $\vec{e}_{q,||}$. Let us consider an electron, which is initially in the state $\psi_{1,k_y}$ of the first level. Absorption of a non-equilibrium phonon results in the transition of this electron into the $\psi_{2,k_y+q_0 e_{q,||}}$ state of the second level. Then the stimulated emission of the phonon with the same $\vec{q}$ transfers the electron back into $\psi_{1,k_y}$. As a result the electron density oscillates between these two states with the Rabi frequency [10]:

$$\omega_R \approx n(q_0)/\tau(\vec{e}_q) \quad ,$$

where $\tau(\vec{e}_q)$ is the time of spontaneous transition of an electron from the state $\psi_{2,k_y+q_0 e_{q,||}}$ into the state $\psi_{1,k_y}$. As in the usual optical Rabi effect one should expect the emission of the phonons at the frequency $\Delta/\hbar$ as well as at two additional frequencies: $\Delta/\hbar \pm \omega_R$. In Fig.1b the frequency $\omega_0 = 1/\tau(\vec{e}_q)$ is shown as a function of level splitting. The dependence has a double peak structure similar to the shape of the dissipative conductivity.

To observe the phonon-induced Rabi splitting the initial distribution of non-equilibrium phonons with the energy cutoff $\Delta_{cut}$ should be created [10], where $\Delta < \Delta_{cut} < \Delta + \hbar\omega_R$ and $n(q > \Delta_{cut}/\hbar s) = 0$. The interaction of such phonons with a 2D electron gas results in appearance of a pulse of the phonons with the energy $\Delta + \hbar\omega_R$, higher than in the initial phonon beam. The length of this pulse is determined by the electron relaxation time, which can be estimated from the dissipative conductivity: $\tau_{rel} \sim e^2/kT\sigma_{xx}$. The effect should be observable if the relaxation time is larger than the period of oscillations of electron between two levels: $\tau_{rel} > 1/\omega_R$.

In conclusion, we have shown that the dissipative conductivity of the magnetically-quantized quasi-2D electron system with the filling factor greater than unity has a double peak structure as a function of magnetic field, when the cyclotron energy is close to the intersubband splitting. Such a system can be considered as an effective two-level system and the peaks in conductivity result from the enhancement of the electron-phonon interaction when the separation between levels becomes close to $\hbar s/l$. A similar double peak structure should also be observed in the non-equilibrium phonon spectroscopy, in which case phonon Rabi sidebands are predicted.

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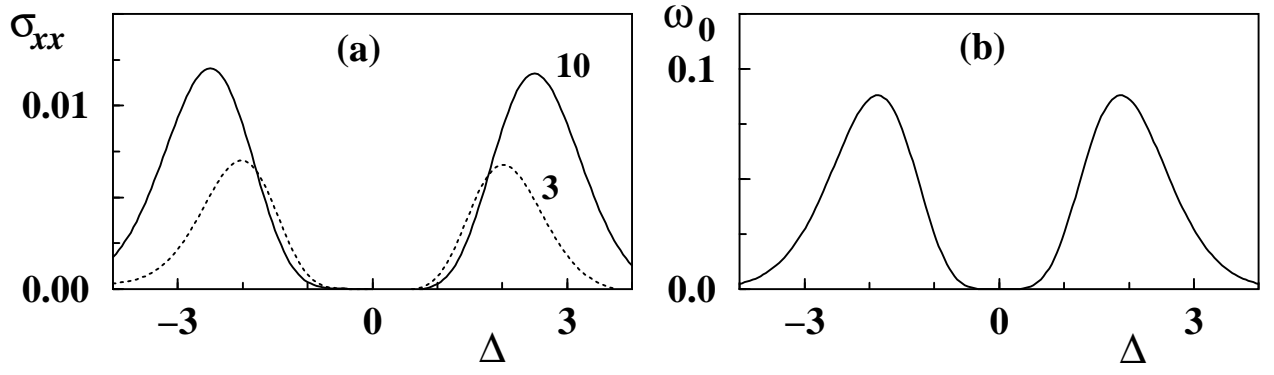
FIG. 1. Dissipative conductivity $\sigma_{xx}$ (a) and frequency $\omega_0 = \frac{1}{\tau(\bar{e}_q)}$ (b) as the functions of level splitting $\Delta$, where $\Delta$ is in units of $\hbar s_1/l$, $\sigma_{xx}$ is in units of $e^2/h\nu$ and $\omega_0$ is in Kelvin. The data are shown for $l(B_0) = 1/b = 50\text{Å}$. The numbers near the curves in (a) show the temperature of the system in Kelvin.